Hydraulic jump in a sloped triangular channel

Mahmoud Debabeche, Sonia Cherhabil, Amin Hafnaoui, and Bachir Achour

Abstract: The hydraulic jump in a sloped triangular channel of 90° central angle is theoretically and experimentally examined. The study aims to determine the effect of the channel's slope on the sequent depth ratio of the jump. A theoretical relation is proposed for the inflow Froude number as function of the sequent depth ratio and the channel slope. An experimental analysis is also proposed to find a better formulation of the obtained relation. For this motive, six positive slopes are tested. The relations obtained are recommended for designing irrigation ditches.

Key words: hydraulic jump, triangular channel, positive slope, open channels, irrigation ditches.

Résumé : Le ressaut hydraulique dans un canal triangulaire a angle d'ouverture de 90° à pente positive est examiné théoriquement et expérimentalement. L'étude a pour but de déterminer l'effet de la pente du canal sur le rapport des hauteurs conjuguées du ressaut. Une relation théorique est proposée exprimant le nombre de Froude incident en fonction du rapport des hauteurs conjuguées et de la pente du canal. Une analyse expérimentale est également proposée afin de trouver une meilleure formulation de la relation obtenue. Pour cela, six pentes positives sont testées. Les relations obtenues sont recommandées pour le dimensionnement des raies d'irrigation.

Mots-clés : ressaut hydraulique, canal triangulaire, pente positive, canaux ouverts, raies d'irrigations.

Introduction

The hydraulic jump is used to dissipate the kinetic energy of a supercritical flow to avoid important modifications of the stilling basin bed. Nevertheless, the triangular section does not satisfy the requirement of a stilling basin, but has some interesting practical applications when used as an irrigation ditch (Achour 1989). The capacity of the hydraulic jump to raise tailwater depth is used to prime a hose siphon designed for the required discharge. The hydraulic jump has been studied by several researchers in recent years. These include Hager and Bretz (1987), Hager (1992), and Ead and Rajaratnam (2002) who studied the hydraulic jump in a horizontal rectangular channel. Hager and Wanoschek (1987), Achour and Debabeche (2003) and Debabeche and Achour (2007), examined the hydraulic jump evolving in a horizontal triangular channel. The first detailed study of the hydraulic jump, in a rectangular channel with positive slope, was by Bakhmeteff and Matzké (1938) who examined the surface profile, the length of the jump, and the velocity distribution. Kindsvater (1944) classified sloped jump according to the position of their toe with regard to the downstream extremity of the slope (Fig. 1), in four types: A-jump for which the toe of the jump coincides with the downstream extremity of the slope, B-jump for which the toe of the jump is between the A-jump and the C-jump, C-jump for which the end of the jump roller coincides with the downstream extremity of the slope, and D-jump for which the jump roller appears completely in the sloped portion. The D-jump was analyzed by Wielogorski and Wilson (1970), Ohashi et al. (1973), Rajaratnam and Muhrahari (1974), Mikhalev and Hoang (1976).

The present study suggests investigating, theoretically and experimentally, the hydraulic jump in a triangular channel with a central angle of 90° and a positive slope. The configuration of the jump adopted for this paper corresponds to the D-jump according to the classification of Kindsvater (1944). The aim of this paper is to propose, for this configuration of jump, a theoretical relation ($F_1 = f(Y, \lambda, \alpha)$) expressing the inflow Froude number F_1 as a function of the angle of inclination α of the channel with regard to the horizontal of the sequent depth ratio ($Y = h_2/h_1$) (h_1 and h_2 are, inflow and final flow depths, respectively) and of the relative length ($\lambda = L_j/h_1$) of the jump. The proposed relation will be obtained by application of the momentum equation applied between the upstream and downstream sections of the jump. In addition, an experimental analysis will be proposed to find a better formulation of the obtained theoretical relation.

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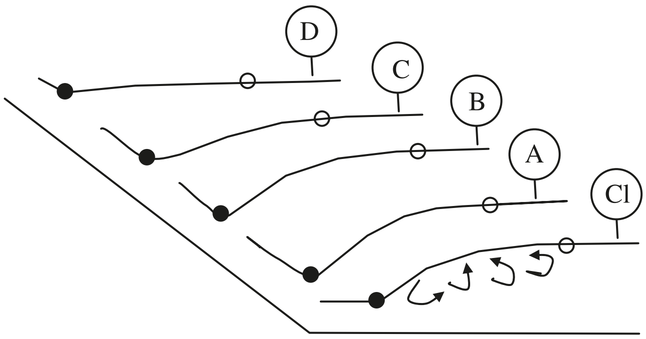
Fig. 1. Classification of sloped jumps according to Kindsvater (1944).



Theory

The momentum equation applied between sections 1 and 2 is written as follows:

$$[1]\qquad \rho Q v_1 + P_1 + W \sin\alpha = \rho Q v_2 + P_2$$

where ρ is the density, and Q is flow discharge.

Figure 2 shows a hydraulic jump evolving in a triangular channel with positive slope. The following hypotheses will be considered in sections 1 and 2: that the pressure is hydrostatic and the friction forces are negligible.

The weight W of the jump and the pressure forces P_1 and P_2 can be expressed by applying the hydrostatics laws as

$$[2]\qquad P_1 = m\varpi \frac{h_1^3 \cos(\alpha)}{3};\ P_2 = m\varpi \frac{h_2^3 \cos(\alpha)}{3};\ W = \varpi V$$

where h_1 and h_2 are flow depths, v_1 and v_2 are average velocities, α is the angle of inclination of the channel, V is the volume of water included between sections 1 and 2,; m is the cotangent of the angle of inclination θ of the channel walls with regard to the horizontal, and ϖ is the specific weight of the fluid. Geometrically, the volume V of the jump in a triangular channel can be deduced from the quarter of a pyramid (Fig. 3*a*).

The geometrical shape of the volume V representative of the hydraulic jump is not perfectly prismatic because of the disturbance of the surface of the jump due to the surface roller. For that purpose, it is necessary to correct this volume by multiplying it by a coefficient k, which is the ratio between the real volume of the jump to the computed volume ABCDEF according to Fig. 3. This coefficient was determined by using experimental data as follows:

$$[3]\qquad V = \frac{1}{3} mkL_j h_1^2 (1 + Y + Y^2)$$

By considering eq. [2] and eq. [3], eq. [1] can be written as

$$[4]\qquad \frac{Q^2}{gmh_1^2} + \frac{mh_1^3 \cos\alpha}{3} + \frac{1}{3} kmL_j h_1^2 (1 + Y + Y^2)\sin\alpha = \frac{Q^2}{gmh_2^2} + \frac{mh_2^3 \cos\alpha}{3}$$

Fig. 2. Hydraulic jump on positive slope.

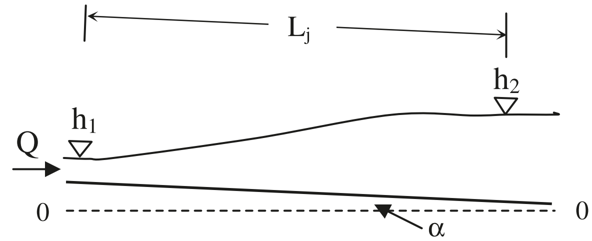
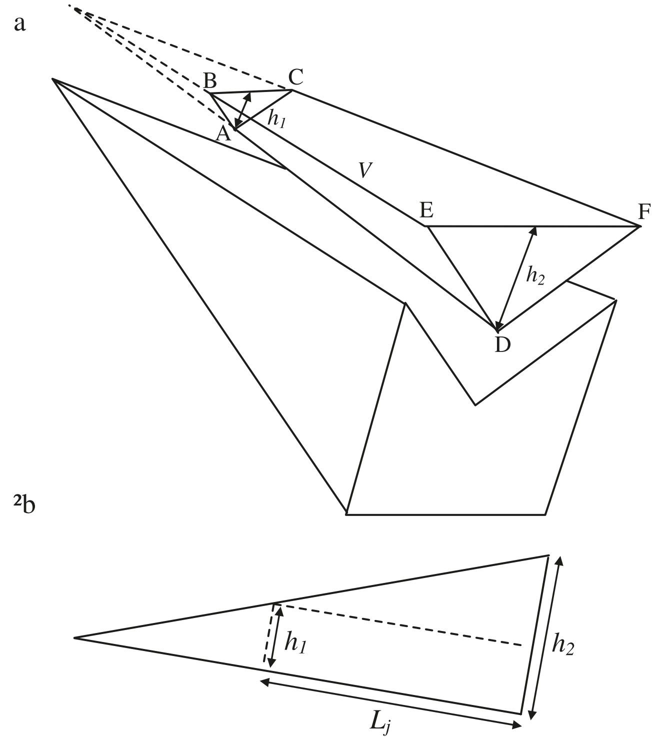


Fig. 3. Geometrical description of the volume of the jump. (*a*) Perspective view. (*b*) Longitudinal section.



The inflow Froude number for a triangular section is written as

$$[5]\qquad F_1 = \sqrt{\frac{2Q^2}{gm^2 h_1^5}}$$

with $m = (\theta)$ (for $\theta = 45°$, $m = 1$), g and θ are, respectively, the acceleration due to gravity and the angle of inclination of the channel walls with regard to the horizontal.

By introducing eq. [5] in eq. [4], one obtains

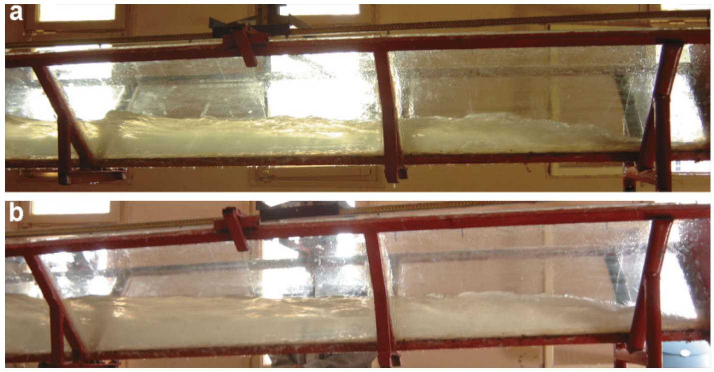
$$[6]\qquad F_1^2 + \frac{2}{3}\cos\alpha + \frac{2}{3} k\lambda (Y^2 + Y + 1)\sin\alpha = \frac{F_1^2}{Y^2} + \frac{2}{3} Y^3 \cos\alpha$$

Equation [6] may be expressed as follows:

$$[7]\qquad F_1^2 = \frac{2}{3} Y^2 \frac{(Y^2 + Y + 1)}{(Y + 1)} \left[\cos(\alpha) - \frac{k\lambda \sin(\alpha)}{Y - 1}\right]$$

Equation [7] expresses the inflow Froude number F_1 as a function of the sequent depths ratio Y of the angle of inclination α of the channel and the relative length $\lambda = L_j/h_1$ of a

Fig. 4. Photograph of the hydraulic jump evolving in a sloped triangular channel. (*a*) F_1 = 8.07, s = 18 cm, L_j = 137 cm, h_2 = 18.25 cm, $tg(\alpha)$ = 0.03. (*b*) F_1 = 6.32, s = 22 cm, L_j = 122 cm, h_2 = 18.82 cm, $tg(\alpha)$ = 0.05.



hydraulic jump with positive slope, evolving in a symmetrical triangular channel of central angle of 90°.

By putting α = 0 in eq. [7], one obtains eq. [8] of Hager and Wanoschek (1987) concerning the classical hydraulic jump in a triangular channel.

$$[8] \qquad F_1^2 = \frac{2}{3} Y^2 \frac{(Y^2 + Y + 1)}{(Y + 1)}$$

To determine the coefficient k of eq. [7], the proposed approach will be analyzed by using experimental data.

From eq. [7] we can also obtain the expression of the relative length of the jump λ:

$$[9] \qquad \lambda = \frac{(Y^3 - 1)\cos(\alpha) - \frac{3F_1^2(Y^2-1)}{2Y^2}}{k(Y^2 + Y + 1)\sin(\alpha)}$$

Experiments

Experiments were made in a symmetrical triangular channel 3 m long with a 90° central angle at the Research Laboratory in Subterranean and Surface Hydraulics (LARHYSS) of the hydraulic department of University of Biskra (Fig. 4). The hydraulic jump was created by setting a sill at the channel extremity. Six positions of the slope were tested so that the tangent of the angle of inclination α with regard to the horizontal takes the following values: 0, 1%, 2%, 3%, 4%, and 5%. For each value of the channel slope, a large range of the inflow Froude numbers was obtained ($3 < F_1 < 13$).

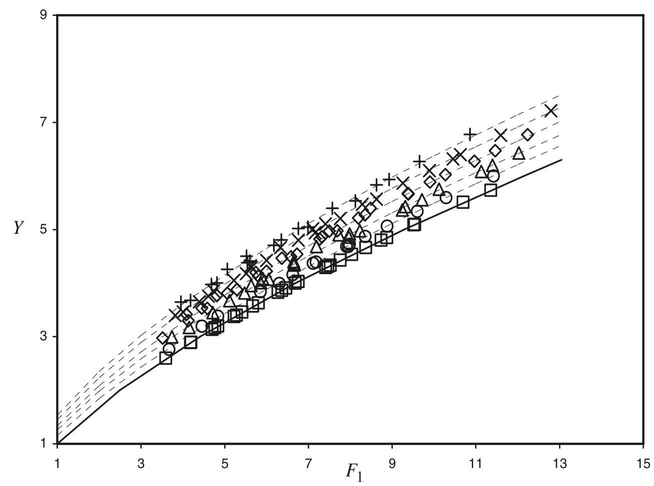
Determination of *k* coefficient

From eq. [7] one obtains the following expression of the coefficient k:

$$[10] \qquad k = \frac{(Y^3 - 1)\cos(\alpha) - \frac{3F_1^2(Y^2-1)}{2Y^2}}{\lambda(Y^2 + Y + 1)\sin(\alpha)}$$

The k value was found by regression by using experimental data and its value is k = 1.12. This coefficient is a constant and does not depend on the slope of the channel. This observation was also verified by McCorcodale and Mohamed (1994) as well as Pagliara and Peruginelli (2000) for the jump evolving in a rectangular channel with adverse slope.

Fig. 5. Variation of the sequent depth ratio Y as function of the inflow Froude number F_1 for a sloped jump according to eq. [7], for six slopes (%): (□) 0 (according to Debabeche and Achour 2007), (*o*) 1, (Δ) 2, (◇) 3, (×) 4 and (+) 5. (—) curve according to eq. [8]. (- - -) curves according to eq. [11].



Explicit relation of the sequent depths ratio $Y(F_1, i)$

One notices that eq. [7] appears under an implicit form with regard to the sequent depths ratio Y and its application consequently requires the use of an iterative process. (A graphic adjustment (Fig. 5) of eq. [7] will make this one explicit in Y.

Figure 5 shows that for a fixed F_1, the tailwater depth h_2 increases with an increase in the channel slope i. By using the experimental data, the analysis of eq. [7] finds the following equation of regression:

$$[11] \qquad Y = (9.15i + 1.07)F_1^{(0.7-1.4i)} \qquad \text{for} \quad 0 \leq i = tg(\alpha) \leq 0.05 \quad \text{and} \quad 3 < F_1 < 13$$

Equation [11] gives a simple means for the determination of the sequent depths ratio Y, using the inflow Froude number F_1 and the channel slope i.

Example of application

The following numerical exampleshows the utility of the obtained relations. By knowing the discharge $Q = 0.5$ m^3/s, the inflow depth $h_1 = 0.3$ m, and the slope $i = 5\%$ of a hydraulic jump in a sloped triangular channel of 90° central angle, the inflow Froude number can be obtained from eq. [5]. ($F_1 = 4.58$); the tailwater depth is obtained from eq. [11] ($h_2 = Yh_1 = 1.19$ m), and the length of the jump is obtained from eq. [9] ($L_j = \lambda h_1 = 8.36$ m).

Conclusion

The hydraulic jump in a sloped triangular channel with a 90° central angle is theoretically and experimentally studied. The configuration of the jump adopted for this paper corresponds to the D-jump type. A general relation is obtained for the inflow Froude number as function of the sequent depths ratio of the angle of inclination of the channel and of the relative length of the jump. The coefficient k, which represents the ratio between the weight of the real volume of the jump to the weight of the computed one, was found by using experimental data and it is a constant that does not depend on the inflow Froude number or the channel slope. However, the obtained relation appears under an implicit form with regard to the sequent depths ratio Y, and an explicit relation is proposed. This relation in particular permits a direct determination of the sequent depth ratio Y, knowing the inflow Froude number F_1, and the slope of the channel.

References

Achour, B. 1989. Jump flowmeter in a channel of triangular cross-section without weir. Journal of Hydraulic Research, **27**(2): 205–214.

Achour, B., and Debabeche, M. 2003. Control of hydraulic jump by sill in a triangular channel. Journal of Hydraulic Research, **41**(3): 97–103.

Bakhmeteff, B.A., and Matzké, A.E. 1938. The hydraulic jump in sloped channel. Transactions, ASME, **60**(HYD-60-1): 111–118.

Debabeche, M., and Achour, B. 2007. Effect of sill in the hydraulic jump in a triangular channel. Journal of Hydraulic Research, **45**(1): 135–139.

Ead, S.A., and Rajaratnam, N. 2002. Hydraulic jump on corrugated beds. Journal of Hydraulic Engineering, **128**(7): 656–663. doi:10.1061/(ASCE)0733-9429(2002)128:7(656).

Hager, W.H. 1992. Energy dissipators and hydraulic jump. Kluwer Academic Publ., Water Science and Technology Library, Dordrecht, The Netherlands. **Vol. 8,** 288 p.

Hager, W.H., and Bretz, N.V. 1987. Hydraulic jumps at positive and negative steps. Journal of Hydraulic Research, **24**(4): 237–253. Discussion **25**(3): 407–413.

Hager, W.H., and Wanoschek, R. 1987. Hydraulic jump in triangular channel. Journal of Hydraulic Research, **25**(5): 549–564.

Kindsvater, C.E. 1944. The hydraulic jump in sloping channels. Transactions ASCE, **109**: 1107–1154.

Mikhalev, M.A., and Hoang, T.A. 1976. Kinematic caracetritics of a hydraulic jump on a sloping apron. Power Technology and Engineering, **10**(7): 686–690.

McCorcodale, J.A., and Mohamed, M.S. 1994. Hydraulic jumps on adverse slope. Journal of Hydraulic Research, **32**(1): 119–130.

Ohashi, I., Sakabe, I., and Aki, S. 1973. Design of combined hydraulic jump and sky-jump energy dissipators of flood spillway. *In* Proceedings of the 13th Congress of ICOLD, Q.41, R.19. pp. 311–333.

Pagliara, S., and Peruginelli, A. 2000. Limiting and sill-controlled adverse-slope hydraulic jump. Journal of Hydraulic Engineering, **126**(11): 847–850. doi:10.1061/(ASCE)0733-9429(2000)126:11(847).

Rajaratnam, N., and Murahari, V. 1974. Flow characteristics of sloping channel jumps. Journal of Hydraulic Engineering, **100**: 731–740.

Wielogorski, W., and Wilson, E.H. 1970. Non-dimensional profile area coefficients for hydraulic jump in sloping rectangular channels. Water Power, **22**(4): 144–150.

List of symbols

F_1 inflow Froude number [–]
g acceleration due to gravity [m·s^{-2}]
h_1 upstream sequent depth [m]
h_2 downstream sequent depth [m]
i channel slope ($i = tg(\alpha)$)
L_j length of jump [m]
k coefficient of correction of the jump weight [–]
m cotangent of the angle of inclination of the channel walls with regard to the horizontal [–]
Q flow discharge [m^3·s^{-1}]
V volume of the jump included between sections 1 and 2 [m^3]
Y sequent depth ratio ($Y = h_2/h_1$) [–]
α angle of inclination of the channel with regard to the horizontal [rad]
θ angle of inclination of the channel walls with regard to the horizontal [rad]
λ relative length of jump ($\lambda = L_j/h_1$) [–]