

# Fuzzy Sliding Mode Control with Chattering Elimination for a Quadrotor Helicopter in Vertical Flight

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**Abstract.** The control of the helicopter includes nonlinearities, uncertainties and external perturbations that should be considered in the design of control laws. This paper presents a control strategy for quadrotor helicopter, based on the coupling of the fuzzy logic control and sliding mode control (SMC), using a nonlinear sliding surface. The main purpose of this work is to eliminate the chattering phenomenon. To achieve our purpose we have used a fuzzy logic control to generate the hitting control signal, the results of our simulations indicate that the control performance of the quadrotor are satisfactory and the proposed fuzzy sliding mode control (FSMC) can achieve favorable tracking performance.

**Keywords:** Sliding mode, Fuzzy Logic, Fuzzy Sliding Mode Control, quadrotor, Dynamic modeling.

## 1 Introduction

Autonomous Unmanned Air vehicles (UAV) are increasingly popular platforms, due to their use in military applications, traffic surveillance, environment exploration, structure inspection, mapping and aerial cinematography, in which risks to pilots are often high. Rotorcraft has an evident advantage over fixed-wing aircraft for various applications because of their vertical landing/take-off capability and payload. Among the rotorcraft, quadrotor helicopters can usually afford a larger payload than conventional helicopters due its four rotors. Moreover, small quadrotor helicopters possess a great maneuverability and are potentially simpler to manufacture. For these advantages, quadrotor helicopters have received much interest in UAV research [1].

The quadrotor is an underactuated system with six outputs and four inputs, and the states are highly coupled, Many efforts have been made to control quadrotor helicopter and some strategies have been developed to solve the path following problems for this type of system, First of this works the quadrotor has been controlled in 3 DOF such as the author in [2] take into account the gyroscopic effects and show that the classical model independent PD controller can stabilize asymptotically the attitude of the quadrotor aircraft. Moreover, they used a new Lyapunov function,

which leads to an exponentially stabilizing controller based upon the PD2 and the compensation of coriolis and gyroscopic torques. While in [3] the authors develop a PID controller in order to stabilize altitude. In [4] a PID controller and a LQ controller were proposed to stabilize the attitude. The PID controller showed the ability to control the attitude in the presence of minor perturbation and the LQ controller provided average results. In [5] the authors the combination of the backstepping technique and a nonlinear robust PI controller. The integral action gain is nonlinear and based on a switching function that ensures a robust behaviour for the overall control law. In [6] they proposed the Backstepping Fuzzy Logic controller (BFL) and Backstepping Least Mean Square controller (BLMS) as new approaches to control the attitude stabilization of quadrotor UAV. And there are many works which control the quadrotor in 6 DOF, First of all, several backstepping and feedback linearization controllers have been developed. In [7] present the nonlinear control techniques applied to an autonomous micro helicopter type Quadrotor using the backstepping approach, In [8] presented the Backstepping Approach for Controlling a quadrotor Using Lagrange Form Dynamics In addition, two neural networks are introduced to estimate the aerodynamic components, one for aerodynamic forces and one for aerodynamic moments. In [9] a mixed robust feedback linearization with linear  $GH_\infty$  controller is applied to a nonlinear quadrotor unmanned aerial vehicle. In [10] the control strategy includes feedback linearization coupled with a PD controller for the translational subsystem and a backstepping-based PID nonlinear controller for the rotational subsystem of the quadrotor. And there is another non linear control technique applied to the quadrotor such as in [11] applied a robust adaptive-fuzzy control. This controller showed a good performance against sinusoidal wind disturbance. In [12] presented the comparison between a based model method and a fuzzy inference system to controlling a drone.

The sliding mode control has been applied extensively to control quadrotors. The advantage of this approach is its insensitivity of the model errors, parametric uncertainties, ability to globally stabilize the system and other disturbances [13]. In [14] author used the sliding mode approach to control a class of underactuated systems (quadrotor), In [15] the authors presents a continuous sliding mode control method based on feedback linearization applied to a Quadrotor UAV, In [7, 16] These papers present a new controller based on backstepping and sliding mode techniques for miniature quadrotor helicopter, In [1] presents two types of nonlinear controllers for an autonomous quadrotor helicopter. The first type is a feedback linearization controller that involves high-order derivative terms and turns out to be quite sensitive to sensor noise as well as modelling uncertainty. The second type involves a new approach to an adaptive sliding mode controller using input augmentation in order to account for the underactuated property of the helicopter. In the literature there are many works in the field of hybrid artificial intelligence systems such as [21, 22, 23]. Our contribution is based on the combination between the sliding mode and fuzzy logic technique (hybrid control law) using the nonlinear sliding surface in order to eliminate the chattering phenomenon. Then, we present a control technique based on the development and the synthesis of a control algorithm based upon sliding mode to

ensure the locally asymptotic stability and the desired tracking trajectories expressed in terms of the center of mass coordinates along (X, Y, Z) axis and yaw angle, while the desired roll and the pitch angles are deduced unlike to [8]. Finally all synthesized control laws are highlighted by simulations which gave results considered to be satisfactory.

## 2 Quadrotors Dynamics Modeling

A quadrotor helicopter is a highly nonlinear, multivariable, strongly coupled, and underactuated system (six degrees of freedom (6 DOF) with only 4 actuators). The main forces and moments acting on the quadrotor are produced by propellers. There are two propellers in the system rotating in opposite direction to balance the total torque of the system. Changing the 2 and 4 propeller’s speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller’s speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers. the quadrotor configuration as shown in Fig. 1.

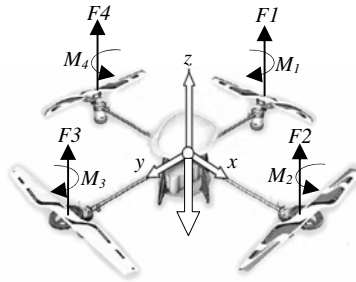


Fig. 1. Quadrotor configuration

The dynamic model presented in [7] includes the gyroscopic effects resulted from both the rigid body rotation in space and the four propulsion groups rotation that can be represented as:

$$\begin{cases}
 \ddot{\phi} = \frac{1}{I_x} \{ \dot{\theta} \dot{\psi} (I_y - I_z) - J_r \bar{\Omega} \dot{\theta} + dU_2 \} \\
 \ddot{\theta} = \frac{1}{I_y} \{ \dot{\phi} \dot{\psi} (I_z - I_x) - J_r \bar{\Omega} \dot{\theta} + dU_3 \} \\
 \ddot{\psi} = \frac{1}{I_z} \{ \dot{\phi} \dot{\theta} (I_x - I_y) + U_4 \} \\
 \ddot{x} = \frac{1}{m} U_1 \{ \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \} \\
 \ddot{y} = \frac{1}{m} U_1 \{ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \} \\
 \ddot{z} = \frac{1}{m} \{ (\cos \phi \cos \theta) U_1 \} - g
 \end{cases} \tag{1}$$

Where  $U_1, U_2, U_3$  and  $U_4$  are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -b & 0 & b \\ b & 0 & -b & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (2)$$

And 
$$\bar{\Omega} = \omega_1 - \omega_2 + \omega_3 - \omega_4 \quad (3)$$

And the parameters of the quadrotor used in dynamic modelling are given in Table 1.

**Table 1.** Physical parameters of the quadrotor

Symbol	Definition
$I_{x,y,z}$	body inertia
$\omega_{1,\dots,4}$	rotor speed
$\phi$	roll angle
$\theta$	pitch angle
$\psi$	yaw angle
$g$	Gravitational acceleration
$m$	Total weight of the quadrotor
$J_r$	rotor inertia
$b$	thrust factor
$d$	drag factor

### 3 Rotor Dynamics

The rotor is a unit constituted by D.C-motor actuating a propeller via a reducer. The D.C-motor is governed by the following dynamic equations:

$$\begin{cases} V = ri + L \frac{di}{dt} + k_e \omega \\ k_m = J_r + C_s + k_r \omega^2 \end{cases} \quad (4)$$

The different parameters of the motor are defined such as:

- $V$  : Motor input
- $k_e, k_m$  : Electrical and mechanical torque constant respectively
- $k_r$  : Load constant torque
- $r$  : Motor internal resistance
- $J_r$  : Rotor inertia
- $C_s$  : Solid friction

Then the model chosen for the rotor is as follows

$$\dot{\omega}_i = bV_i - \beta_0 - \beta_1 \omega_i - \beta_2 \omega_i^2 \quad i \in [1,4] \quad (5)$$

With 
$$\beta_0 = \frac{C_s}{J_r}, \beta_1 = \frac{k_e k_m}{r J_r}, \beta_2 = \frac{k_r}{J_r} \text{ and } b = \frac{k_m}{r J_r}$$

## 4 Control Strategy

To achieve a robust path following for the quadrotor helicopter, two techniques, capable to control the helicopter in presence of sustained external disturbances, parametric uncertainties and unmodelled dynamics, are combined. The proposed control strategy is based on the decentralized structure of the quadrotor helicopter system, which is composed of the dynamic Equation (1). The overall scheme of the control strategy is depicted in Fig. 2. The translational motion control is performed in two stages. In the first one, the helicopter height,  $z$ , is controlled and the total thrust,  $U_1$ , is the manipulated signal. In the second stage, the reference of pitch and roll angles ( $\theta_r$  and  $\phi_r$ , respectively) are generated through the two virtual inputs  $U_x$  and  $U_y$ , computed to follow the desired  $xy$  movement. Finally the rotation controller is used to stabilize the quadrotor under near quasi-stationary conditions with control inputs  $U_2, U_3, U_4$ .

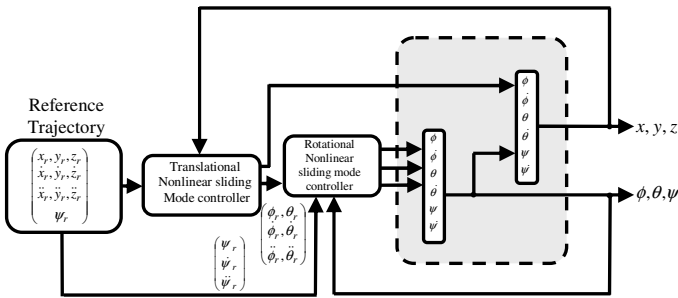


Fig. 2. Quadrotor helicopter control structure

## 5 Sliding Mode Control Using Non Linear Sliding Surface

In this section, the first goal in our topic is to characterize a class of manifold on which the control objective is achieved. We recall that the sliding mode control objective consist of first designing a suitable manifold  $\psi(x, t) \in \mathfrak{R}^m$  defined by  $\psi = \{x \in \mathfrak{R}^n / S(x) = 0\}$ , to restrict state trajectories of the plant to this manifolds to result in the desired behaviors such as tracking, regulation, and stability then, determining a switching control law  $u(x, t)$ , which is able to drive the state trajectory to this manifolds and maintain it on this manifolds for all the time that is  $u(x, t)$  is determined such that the selected manifold  $\psi(x, t)$  is made attractive and invariant. Generally in sliding mode control, the switching surfaces are linear functions. Slotine and Li [17] gave a form of this sliding manifold which was a Hurwitz polynomial of the error and its derivative were up to  $(r-1)$ , where  $r$  is the relative degree of the output. For SMC using linear elements, the linear switching surfaces often yield a satisfactory systems response, in terms of stability and robustness, to the parameter variation and disturbances. However, with linear switching surface the speed of

transient response is relatively slow [18]. In this paper, the design and analysis of SMC with nonlinear switching surface are considered.

From the fact that the output  $Y = [x, y, z, \phi, \theta, \psi]^T$ , where the relative degree is two for each subsystem, in order to obtain static feedback, we define the manifolds  $\psi(e)$  as follows:

$$\psi(e) = \left\{ e \in \mathfrak{R}^n / S(e) = \dot{e} + \Lambda(e) = 0 \right\} \tag{6}$$

With  $e = Y - Y_d$  is the tracking error,  $\Lambda(\cdot)$  is any given  $C^1$  function whose property will be derived below. One has following result.

**5.1 Proposition 1**

Consider the manifolds  $\psi(e)$  in (6) and assume that  $\Lambda(\cdot)$  is a continues function such that  $e\Lambda(e) > 0, \forall e \neq 0$ . Then on the manifolds  $\psi$ , the output error  $e$  converges at least asymptotically to zero.

**Proof:** Due to the manifolds  $\psi$  we have

$$\dot{e} = -\Lambda(e) \tag{7}$$

Let use the Lyapunov function given by  $V = \frac{1}{2} e^2$ . Its derivative is then

$$\dot{V} = -e\Lambda(e) \tag{8}$$

In order to make  $\dot{V}$  negative definite, it is enough that  $e\Lambda(e) > 0, \forall e \neq 0$ . Hence, the outout error  $e$  is bounded and moreover, it tends asymptotically to zero.

Hence  $\psi$  are suitable manifolds for our control system, since the control objective is achieved on it. Let us now design the control low  $u$  that make,  $\psi$  attractive and invariant. The function  $\Lambda(\cdot)$  is given in (9) and characterized in proposition 1. Then,  $\psi$  is globally attractive and invariant.

$$\begin{cases} \Lambda(e) = \frac{2}{1 + e^{-\mu e}} - 1 & \mu > 0 \\ \frac{d\Lambda(e)}{de} = \frac{\mu}{2} [1 - \Lambda(e)^2] \end{cases} \tag{9}$$

The desired roll and pitch angles in terms of errors between actual and desired speeds are, thus, separately given by:

$$\phi_r = \arcsin (U_x \sin \psi - U_y \cos \psi) \tag{10}$$

$$\theta_r = \arcsin \left( \frac{U_x}{\cos \phi \cos \psi} - \frac{\sin \phi \sin \psi}{\cos \phi \cos \psi} \right) \tag{11}$$

### 6 Fuzzy Sliding Mode Controller Design

The conventional sliding mode controller results in high frequency oscillations in its outputs, causing a problem known as chattering. The chattering is undesirable because it can excite the high frequency dynamics of the system. To eliminate chattering, a continuous fuzzy logic control  $\Delta u_{fuzzy-slid}$  is used to approximate the discontinues control. The design of the fuzzy controller begins with extending the crisp sliding surface  $S = 0$  to the fuzzy sliding surface defined by linguistic expression [18]:

$$\tilde{s} \text{ is zero} \tag{12}$$

Where  $\tilde{s}$  is the linguistic variable for S and ZERO is one of its fuzzy sets. In order to partition the universe of discourse of S, the following fuzzy sets are introduced.

$$T(\tilde{s}) = \{NB, NM, ZE, PM, PB\} = \{F_s^1, \dots, F_s^5\} \tag{13}$$

where  $T(\tilde{s})$  is the term set of  $\tilde{s}$ , and NB, NM, ZR, PM, and PB are labels of fuzzy sets, which are negative big, negative medium, zero, positive medium, and positive big, respectively. For the control output  $\Delta u_{fuzzy-slid}$ , its term set and labels of the fuzzy sets are defined similarly by

$$T(\tilde{u}_s) = \{NB, NM, ZE, PM, PB\} = \{F_u^1, \dots, F_u^5\} \tag{14}$$

The membership functions of these fuzzy sets are depicted in Fig. 3. In Fig. 3 (a),  $r \in [0, 1]$  is a coefficient to be used to adjust the input centre point, and  $\Phi$  is the defined boundary layer around the switch surface.

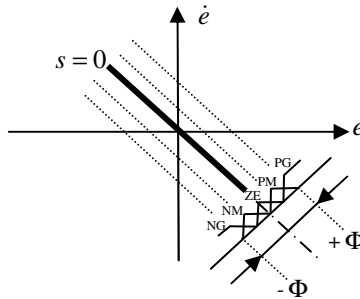


Fig. 3. Fuzzy partition of the space around the sliding surface

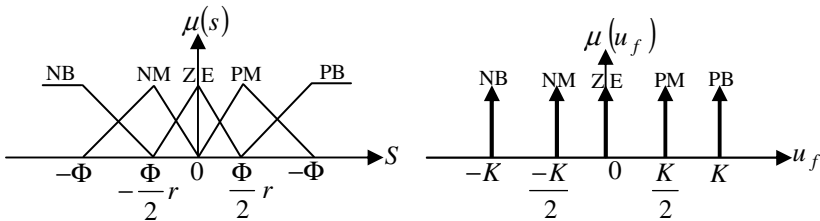


Fig. 4. Representation of term sets,  $T(\tilde{s})$  and  $T(\tilde{u}_s)$

From these two term sets, we can build the following fuzzy rules [19]:

- R1 : IF  $s$  est NB Than  $\Delta u_{fuzzy-slid}$  is PB    R2 : IF  $s$  est NM Than  $\Delta u_{fuzzy-slid}$  is PM  
 R3 : IF  $s$  est ZE Than  $\Delta u_{fuzzy-slid}$  is ZE    R4 : IF  $s$  est PM Than  $\Delta u_{fuzzy-slid}$  is NM  
 R5 : IF  $s$  est PB Than  $\Delta u_{fuzzy-slid}$  is NB

Once the membership functions and fuzzy rules are determined, the final step is the defuzzification, which is the procedure to determine a crisp control for  $\Delta u_{fuzzy-slid}$ . There are many defuzzification strategies such as the maximum criterion, the mean of maximum, the centre of area, and the weighted average method [18, 19]. We use the weighted average method to get the crisp control for  $\Delta u_{fuzzy-slid}$ . Then

$$\Delta u_{fuzzy-slid} = \frac{\sum_{i=1}^5 C_{fi} \mu_i (s)}{\sum_{i=1}^5 \mu_i (s)} \tag{15}$$

Where  $C_{fi}$  is the associated singleton membership function of  $\Delta u_{fuzzy-slid}$ .

### 7 The Proposed Fuzzy Sliding Mode Control of the Quadrotor

In this section, the objective is to apply the hybrid fuzzy sliding mode control in order to solve the problem of chattering phenomenon.

The control system applied to the quadrotor is given by:

$$u = u_{eq} + \Delta u_{fuzzy-slid} \tag{16}$$

With the discontinued control  $\Delta u_{fuzzy-slid}$  is calculated by a fuzzy inference system, its description is already given in section 6.

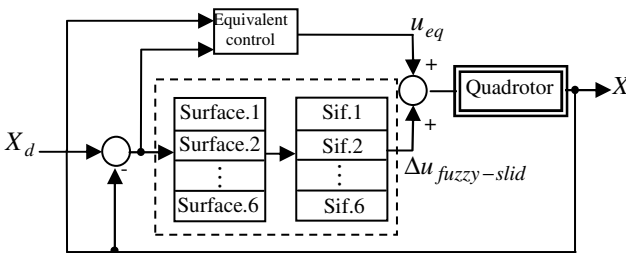


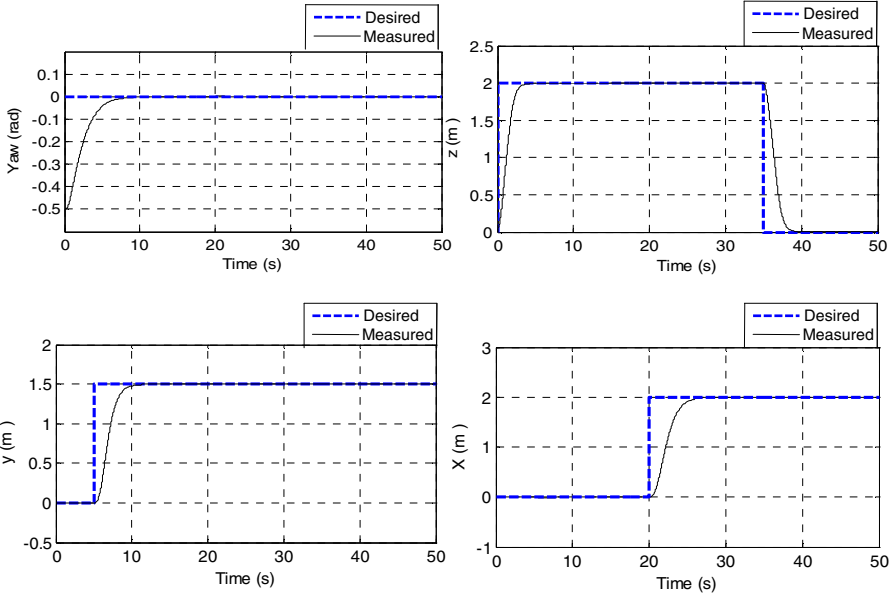
Fig. 5. Block diagram of the fuzzy-sliding control system applied to quadrotor

### 8 Simulation Results

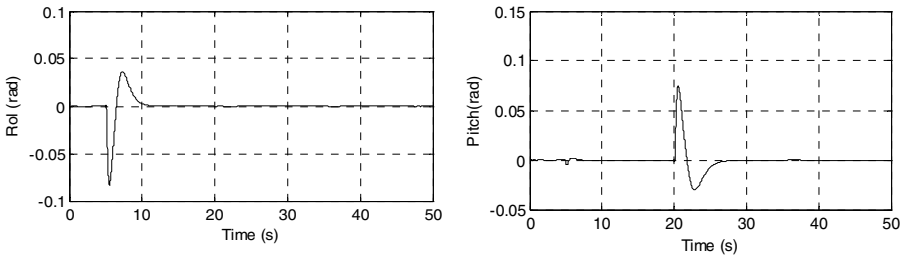
This section presents the simulation results of the position control of quadrotor helicopter. The simulation results are presented in Fig.6, Fig.7, Fig.8, Fig.9, and Fig.10 we can see that, the controller ensures a good tracking performance, The control



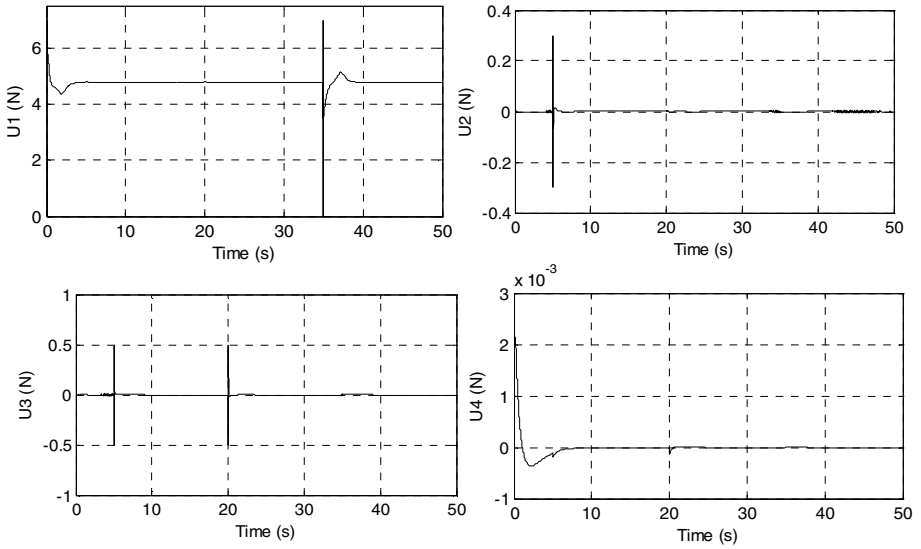
efforts applied to the system are presented in the Fig.8 we remark the elimination of chattering problem permits the smoothness of the control law, The propellers speeds are given in Fig.9 where we can see their stabilization at the value 200 (radian/sec) in a finite time. Our approach (fuzzy sliding mode using non linear surface) presented better results in comparison to another works such as [10, 20].



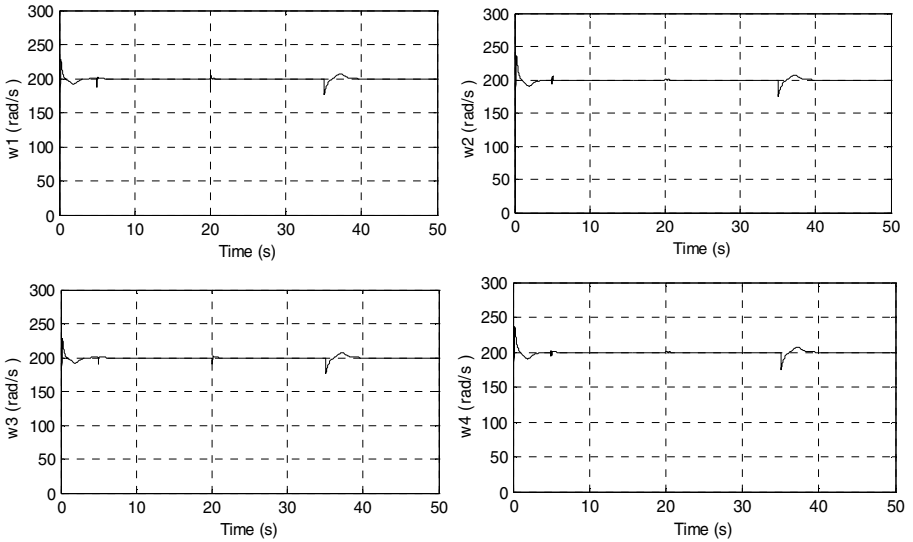
**Fig. 6.** Tracking simulation results of the desired trajectories along yaw angle ( $\psi$ ) and the ( $X, Y, Z$ ) axis using fuzzy Sliding Mode with nonlinear sliding surface



**Fig. 7.** Simulation results of the roll angle ( $\phi$ ) and the pitch angle ( $\theta$ ) using fuzzy sliding mode with nonlinear sliding surface



**Fig. 8.** Control response of a quadrotor helicopter using fuzzy sliding Mode with nonlinear sliding surface



**Fig. 9.** Angular velocities of the four rotors using fuzzy sliding mode with nonlinear sliding surface

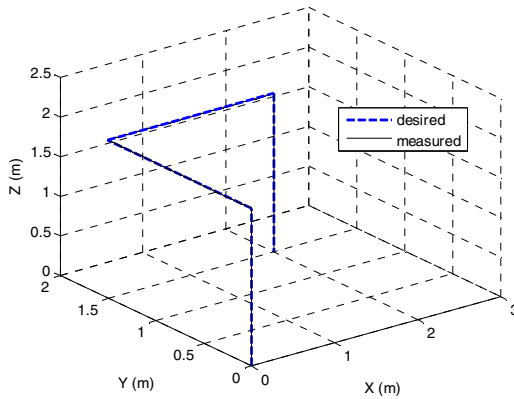


Fig. 10. Global trajectory of the quadrotor in 3D using fuzzy Sliding Mode

## 9 Conclusion

In this work, we addressed the position control problem of the helicopter's quadrotor taking into account the dynamics. A solution based on the fuzzy sliding mode control method using non linear sliding surface is proposed. This method is a combination of fuzzy logic control and the conventional sliding mode control, called fuzzy sliding mode control. This combination forces the real position towards the values required to achieve the control objective. Through the simulation results, we can see that fuzzy logic control can be applied to reduce the chattering of the sliding mode control.

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